

AD614595

(FINAL REPORT)

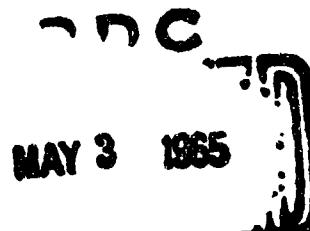
RESEARCH AND DEVELOPMENT FOR QUANTIFYING PROCESSING
OF
PROBABILIS . (UNCERTAIN) DATA

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-65-162

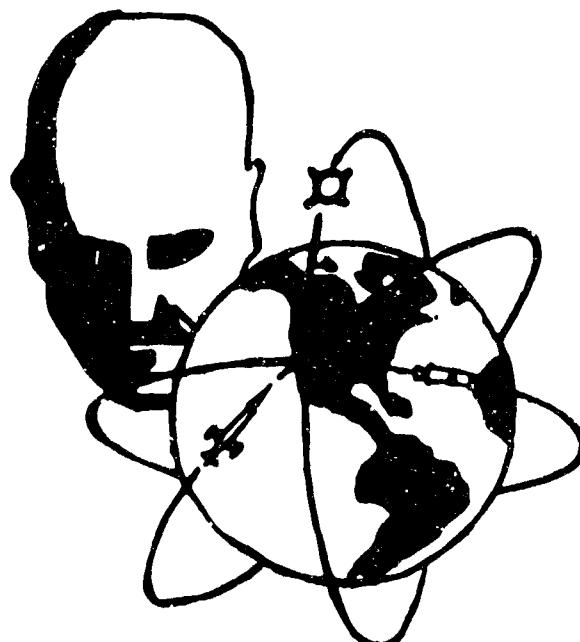
DECEMBER 1964

William H. Wynn

COPY	OF	3	THE
HARD COPY		\$. 2 . 00	
MICROFICHE		\$. 0 . 50	



DECISION SCIENCES LABORATORY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts



ARCHIVE COPY

Project 4690, Task 469003

(Prepared under Contract No. AF 19 (628)-1610 by the University of North Carolina,
Psychometric Laboratory, Chapel Hill, North Carolina.)

DDC AVAILABILITY NOTICE

Qualified requesters may obtain copies from Defense Documentation Center (DDC). Orders will be expedited if placed through the librarian or other person designated to request documents from DDC.

DISSEMINATION NOTICE

Copies available at Office of Technical Services, Department of Commerce.

LEGAL NOTICE

When US Government drawings, specifications or other data are used for any purpose other than a definitely related government procurement operation, the government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

OTHER NOTICES

Do not return this copy. Retain or destroy.

ESD-TDR-65-162

(FINAL REPORT)

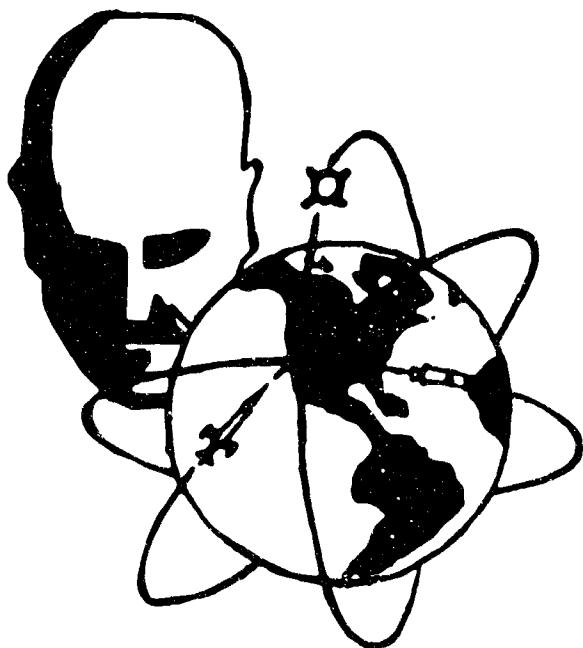
RESEARCH AND DEVELOPMENT FOR QUANTIFYING PROCESSING
OF
PROBABILISTIC (UNCERTAIN) DATA

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-65-162

DECEMBER 1964

William H. Wynn

DECISION SCIENCES LABORATORY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts



Project 4690, Task 469003

(Prepared under Contract No. AF 19 (628)-1610 by the University of North Carolina,
Psychometric Laboratory, Chapel Hill, North Carolina.)

ABSTRACT

This report describes research in decision making, with emphasis on the quantification of processing of probabilistic (uncertain) data, undertaken in the Psychometric Laboratory, University of North Carolina, from 1962 to 1964. Section I of this report gives relevant administrative information; Section II describes the general objectives and reviews findings of the research; Section III provides abstracts of papers reporting research conducted under Contract AF19(628)-1610. This contract was monitored by Dr. Anne Story of the Decision Sciences Laboratory.

This Technical Report has been reviewed and approved.

Donald W. Connolly
DONALD W. CONNOLLY
Acting Chief, Decision Techniques Division
Decision Sciences Laboratory

Roy Morgan
ROY MORGAN
Colonel, USAF
Director, Decision Sciences
Laboratory

CONTENTS

I.	Administrative Information	1
II.	General Review of Research Objectives and Findings	2
III.	Abstracts of Technical Documentary Reports	19

I. ADMINISTRATIVE INFORMATION

The research project reviewed in this report was supported by the Decision Sciences Laboratory, Electronic Systems Division, U. S. Air Force, under Contract No. AF 19(628)-1610. The project began in the summer of 1962 and terminated in 1964. Dr. Emir H. Shuford served as principal investigator during the earlier portion of the project; Dr. Albert H. Amon served as principal investigator during the latter portion. Other participants in the project have included Dr. Masanao Toda, Mr. William H. Wynn, Mr. Amnon Rapoport, Mr. David Messick, Mr. Edward Massengill, Mr. Raymond Wiesen, and Mr. Larry Gordon. Dr. Lyle V. Jones has provided advisory assistance throughout the course of the research program. This final report has been prepared by Mr. William Wynn.

II. GENERAL REVIEW OF RESEARCH OBJECTIVES AND FINDINGS

Most of the research undertaken in this project attempted to show how Bayesian decision theory might be used as a descriptive theory of human information processing. There are several reasons for using decision theory as a starting point for a theory of human behavior. The first reason is that a decision-theoretical analysis of a task yields an upper limit on the performance possible in the task as presently structured. This upper limit of performance is relevant in evaluating the efficiency of human operators and the adequacy of training programs, as well as guiding the development of man-machine systems which yield maximal performance. Another reason for using decision theory as a descriptive theory of human behavior derives from the fact that it is a model which represents optimally efficient behavior. It usually proves an easy task to modify this model to correspond to the apparent relative inefficiency of behavior by introducing certain constraints such as a limited and imperfect memory, information processing errors, "cost of thinking," etc. The validity of these modifications can be empirically determined.

There are a number of possible decision-theoretic formulations. They all share in common the notion of a payoff matrix in which a) the rows represent the alternative courses of action available to the decision maker, b) the columns represent either the alternative courses of action available to the opponent in game theory or the possible states of nature in decision making under uncertainty, i.e., statistics broadly interpreted (Luce & Raiffa, 1957; Raiffa & Schlaiffer, 1961), and c) the entries in the cells of the matrix represent the utilities associated with the possible outcomes. The utilities are measured on a psychological continuum defined up to a linear transformation (von Neumann & Morgenstern, 1947; Luce & Raiffa, 1957; Shuford, Jones, & Bock, 1961).

The decision-theoretic formulations differ, however, with respect to the criterion for an optimal strategy (Luce & Raiffa, 1957). It often has been assumed that the minimax strategy--i.e., choose that course of action which results in maximum utility assuming the worst possible outcome--is reasonable at least for the case in which one is playing a strictly competitive game against a rational opponent. This view is subject to the criticism that even in a competitive situation one should take account of the past behavior of the opponent and all other possible relevant information, a policy which is in general incompatible with the minimax strategy, but which is, in fact, an intuitive restatement of a Bayes strategy--i.e., use all available information to assign (subjective or personal) probabilities to the opponent's actions or to the states of nature, use these probabilities to compute the expected utility for each alternative course of action, and then select that course of action which has the largest expected utility.

It seems apparent that Bayesian decision theory is more powerful and more general than the minimax strategy because it includes an additional psychological variable, subjective probability. Also, with the development of a theory of subjective probability (Shuford, 1959; Wiesen & Shuford, 1961) which can account for the modification of probabilities on the basis of past experience and new information, we have a descriptive decision-theoretic model which is dynamic in the sense of accounting for changes in behavior over time. Further, the model describes and can be used to define adaptive behavior.

In developing and testing Bayesian models for individual decision making, the subject's task in an experiment is conceived to be a problem in decision making under uncertainty in which the subject responds on each trial of the experiment in such a way as to maximize his average payoff during the course of the experiment. Two sources of information are available to the subject: a) the information obtained from observing the particular stimulus presented on that trial, and b) the background information as to the probabilities with which the different stimulus values might be presented. This background information is provided by the subject's experience before participating in the experiment, the instructions and the context of the experiment, and during the course of the experiment by the stimuli previously observed by the subject. The background or prior information is available at the beginning of the trial. The subject observes the stimulus and obtains (imperfect) information as to the value of the stimulus on this particular trial. The subject, in effect, combines this specific information with his prior information and obtains a posterior probability distribution (representing the subjective probability of the stimulus value given the observation) which is used to determine the expected utilities of the different response categories and, thus, the optimum response. The models use Bayes' theorem in the computation of the posterior probabilities and in many cases are formally analogous to the models used in Bayesian statistical inference (Schlaifer, 1959; Schlaifer, 1961; Raiffa & Schlaifer, 1961; Shuford, 1961b).

By assuming that man is a Bayesian information processor, several predictions were derived relating probability of correct response, amount of information requested, and decision time to two independent variables of interest--number of response categories and disparity between response alternatives. It was shown that these relations depend upon other aspects of the decision task such as the size of the payoffs, the cost of information, the probabilities of the states, and the conditional probabilities of the data.

During the early portion of the project a number of significant theoretical developments occurred. One line of development carried on by Dr. Masanao Toda and by Mr. David Messick was concerned with the measurement of subjective probability distributions (see Working Papers Nos. 1, 2, 3, 6, and 7). Another line of development carried on by Dr. Emir H. Shuford and by Mr. Raymond

A. Wiesen was concerned with optimal models for learning prior probabilities (see Working Papers Nos. 5, 8, 10, and 18). Dr. Masanao Toda compared Bayesian and minimax strategies in zero-sum two-person games (see Working Paper No. 9). Mr. David Messick applied concepts from Bayesian decision theory and the theory of non-zero-sum games to the social psychological problem of group decision making and group problem solving (Messick, 1963). Dr. Emir Shuford showed how it was possible to take a well-defined decision situation (one in which all distribution functions and parameters are known) and successively degrade the situation down to a point where no decision problem remains, by progressively relaxing the assumptions about the nature of the decision situation (see Working Papers Nos. 15, 17, 19, 21, and 31).

An early experiment performed by Mr. Raymond Wiesen applied decision theory in an investigation of some probabilistic learning situations (Wiesen, 1962). Three related choice situations were presented. The first situation was a standard probabilistic discrimination learning task. Each trial began with the presentation of one of a set of stimuli. The subject chose between the two response alternatives to predict which of two events would occur on the trial, the probability of each event being a function of the stimulus presented. In the second situation, the probabilities of the stimuli given the events, i.e., the conditional probabilities, were introduced to the subject at the beginning of the experiment. The third situation was like the second except that the subject was not told which event occurred on each trial. The decision-theoretic analysis showed what differences in performance would be expected among the three conditions when a strategy which maximizes average expected payoff is employed. One group of subjects was run in each situation with the overall relative frequency of one event equal to .80. The performance of subjects in the first and second situations was virtually identical, indicating that subjects in the second group did not integrate information concerning the overall relative frequencies of events and conditional probabilities. Subjects in the third situation showed poorer performance than those in the other two groups, with a large proportion of these (non-feedback) subjects making every choice in agreement with the assumption that the overall relative frequency of one event was one-half.

As part of the program for the experimental evaluation of predictions from the Bayesian model, two computer-controlled decision tasks were developed. The first task, programmed for the LGP-30 computer of the Psychometric Laboratory by Mr. Amnon Rapoport (see Working Papers Nos. 11 and 26) was a computerized version of a task (see Shuford, Wiesen, Massengill, and Nishisato, "Pre-decisional Processes Related to Psychophysical Judgment" Report No. 33, Psychometric Laboratory, January 1963 for description of original task) requiring estimation of proportions. In this task the indi-

vidual observes a sample of elements from a population and, on the basis of this incomplete information, estimates the percentage of elements of a given type in the population. The individual is paid a fixed amount for estimating the actual percentages within one or two percentage points. Since the distribution of actual percentage occurring during the experiment may deviate markedly from the uniform, a Bayesian decision maker would improve his performance by learning this distribution and combining this information with the sample information to obtain a better estimate. Further, the distribution of actual percentages may change one or more times during the course of an experiment. Thus the Bayesian decision maker must be able to detect these changes and realize that he is operating in a different environment. This computer-controlled task was used to study how humans learn prior probabilities and how they deal with this particular type of nonstationary environment.

In one such study utilizing this computer-controlled task, Mr. Amnon Rapoport (1963) obtained three types of measures of each subject, from which the prior (subjective) probability distribution could be inferred for each block of ten trials. The stimuli in the experiment consisted of a series of 3×3 matrices composed of randomly placed 0's and 1's. Every 3×3 matrix was a sample from a larger 10×10 matrix containing a given proportion of 1's, this proportion being randomly selected from a given beta distribution. The experiment consisted of 120 trials, and the environment was nonstationary, for different beta distributions were used in generating the stimuli for trials 1-50, trials 50-90, and trials 91-120. Subjects not being told that these environmental shifts would occur. On each trial the subject first made a prediction of the proportion of 1's in the large unobservable 10×10 matrix. Then he was presented with the 3×3 matrix, from which he made an estimate of the proportion of 1's in the larger matrix. After each block of ten trials, direct measures of the subjective probability distribution (SPD) were obtained from the subject's responses to three hypothetical bets concerning the value of the next proportion that the computer might select. After making their estimates of the proportion, subjects were informed of the correctness of their estimates and predictions on that trial and of their cumulative earnings, subjects thus being provided with complete feedback on each trial.

Several findings were noted in the experiment, as follows:

- 1) as predicted from the Bayesian model, the subjects' predictions tended to approach the mode of the stimulus distributions, and the variance among subjects decreased as more information was obtained;
- 2) subjects tended to overestimate the variance of the stimulus distribution, indicating that the SPD's were somewhat flatter than the actual stimulus distributions;
- 3) correspondence among the three measures--i.e., predictions, estimates, and responses to hypothetical bets--averaged over subjects, was good, though far from perfect;
- 4) though the beta distribution from which the stimuli were generated changed twice during the experiment, the second shift was responded to much more rapidly than the first, showing that subjects became sensitive to the nonstationarity of

the environment. The Bayesian model was found to be incomplete in that it predicted only the asymptotic performance of the subjects, having nothing to say concerning the learning which leads to this asymptotic state. An extension of the Bayesian model was formulated with the intention of accounting for the trial-to-trial changes in the predictions and estimates of individual subjects. This model had two free parameters, concerning individual differences, one for weight given to past experience and another for the weight given to the successive most recent observations. When values of these parameters providing the best fit for average predictions and estimates of the subjects were used, the fit was remarkably good.

A second study utilizing this same computer-controlled task (Rapoport, 1964) had as its main purpose the testing of a method of measuring the parameters of a continuous SPD, obtained at several points within a sequential decision task. This method, called the Range Betting Method, was developed by Toda (see Working Papers Nos. 1, 2, 3, and 6 and also Toda, 1963). Subjects made predictions and estimates of proportions for 120 trials in a nonstationary environment, as before. This experiment differed from the above in that the stimulus samples consisted of a varying number of 1's and 0's, the number of elements being randomly determined, between 1 and 20, on each trial. Subjects were divided into two groups, one group receiving complete feedback (Group CF), and the second group receiving only partial feedback (Group PF). Subjects in the CF condition were informed on each trial as to correctness of their estimates, the correct value for the proportion being printed by the Flexowriter. Subjects in the PF condition received no information after each trial, but were informed of their earnings only after each block of five trials, thus being given only a general idea as to performance level without knowledge of whether their gains were a function of predictions or estimates or on which trials they were correct. After each block of five trials, subjects in both groups made predictions concerning the range within which the next selected proportion would fall. In comparing the predictions of subjects in this experiment with those in the previous, it was found that subjects in the CF condition gave approximately the same values as those in the previous study, whereas subjects in the PF condition gave predictions significantly different from those in the previous study, showing almost no evidence of learning and with their predictions tending to concentrate near .5. As in the previous experiment, the inferred SPD, averaged over subjects, was flatter than the actual distribution. The subjects' estimates of the range boundaries were compared to predicted values by the range betting method and also to predicted values by an alternative confidence limits' model. The range betting method was found to be adequate for measuring the parameters of a continuous SPD so long as the assumption is made that the SPD is a beta distribution. As this assumption is questionable, the alternative confidence limits model, which also takes account of individual difference parameter concerning the recency of information used in making the range estimates, was considered a more realistic psychological model.

A second computer-controlled decision task devised for experimental testing of the proposed models was that programmed by Mr. Raymond Wiesen (see Working Paper No. 12). In this task, the decision maker must choose one from a small number of alternatives. A fixed payoff is received for the choice of the correct alternative. The decision maker may request information to aid in making the decision, but the more information requested the greater is the cost to the decision maker. The task may be set up in two modes. In the fixed sampling mode, the decision maker states the number of observations desired, this information is provided, and the decision maker must then make his decision. In the sequential sampling mode, the decision maker may request one observation, observe the result, decide to take another observation or to stop and make his terminal decision, and so on. This task is arranged so that cost of information, payoffs, number of alternatives, symmetry or asymmetry of alternatives, prior probabilities, and sampling mode may be varied during an experimental session according to any prearranged schedule. The dependent variables of interest are amount of information requested, earnings, objective expected payoff of actual choice, and objective probability of correctness of actual choice.

An experiment utilizing this computer-controlled task was that of Mr. David Messick (1964a). This experiment in sequential information seeking involved two independent variables: number of terminal acts or response alternatives and information available to the decision maker prior to beginning the task. According to Bayesian decision theory, the larger the number of possible terminal acts, the more pre-decisional information is needed; and the more prior information available about which of the terminal acts is correct, the less current information will be needed to be observed prior to making the decision. On the basis of samples from a binomial distribution, subjects were to estimate the proportion of 1's, or "top quality" items. Subjects sampled the universe sequentially, stopping after some number of observations had been taken in order to make a terminal decision by selecting that one of the mutually exclusive and exhaustive subsets of the unit interval which was believed to contain the proportion. The number of subsets, or possible terminal acts, was either three (.00 to .33, .33 to .67, or .67 to 1.00) or five (.00 to .20, .20 to .40, .40 to .60, .60 to .80, .80 to 1.00). Two prior frequency distributions were used--either a rectangular distribution or a negatively skewed distribution over the unit interval. Thus four different conditions for the 32 experimental trials were possible, defined by the four combinations of the two decision partitions of the unit interval and the two prior frequency distributions. It was found, as expected, that more observations were taken in the 5-act case than in the 3-act case. No effect, however, of prior frequency distributions was found, but there was a tendency for subjects to make more observations in the latter 16 trials than in the first 16. Subjects did not make full use of the information provided by the prior distribution, for fewer observations should have been required when the prior distribution was skewed, since such a

distribution makes for less stimulus uncertainty than does the rectangular distribution.

As no formal model was applicable to the problem presented in this experiment, an optimal Bayes strategy model was derived so as to permit a more detailed investigation of the extent to which human information seeking may be accounted for in terms of a "rational" theory of behavior (Messick, 1964b). The important variable in determining a stopping rule in this task was found to be the expected probability of being correct. A "no information" theorem was presented which shows that under some circumstances when a success or failure on a given trial are equally probable, the probability of being correct after making the observation is identical to the probability of being correct before the observation was taken. A derivation of the Beta-binomial probability function was given which suggested a more tractable computational procedure for the distribution and which illuminates its limiting distribution.

During the course of research undertaken as part of this project, certain limitations in the Bayesian approach to describing human decision processes were noted. The Bayesian model was found to be incomplete, in that it predicts behavior in a situation in which the stimulus distribution is known to the subject, but it has little or nothing to say concerning the process by which the stimulus distribution is learned. Bayesian models may assume unlimited memory on the part of the subjects and may give equal weight to all events observed without regard to recency; full utilization of all observations, as well as accurate integration of current observations with prior information, is also assumed. Much of the research reported in this project was directed toward making extensions of the more elementary Bayesian model and taking account of assumptions of the model which do not hold in human information processing. A central phase of the latter period of the project was the development, by Dr. Albert H. Amon, of a descriptive syntax for general classes of decision structures and the utilization of this analysis in the construction of models of human decision behavior in difficult environments. This theoretical development derived from information processing theory and from experiments in human perception, rather than from the tradition of statistical decision theory; it follows up some lines of thought in his "Decision Structures in Recognition" (Amon, 1962). Because of the importance of errors and other nonoptimal behavior for efficient constriction of the class of alternative models, decision making under stress was emphasized in this undertaking. Dr. Amon had made considerable progress in developing several alternative models, and had begun writing what would have been a monograph to be distributed under terms of the contract at the time of his unexpected death on August 9, 1964. Though extensive notes for this monograph are available, the material is far from being in finished form. The task of organizing and editing this material has been undertaken, however, and it is hoped that a report of Dr. Amon's thinking in this area will be published

at some later time.

Other research within the information processing framework was carried out by Mr. William Wynn, toward construction of a model of certain perceptual and memory aspects of paired-associate learning.

Another activity initiated by Dr. Amon was the design and assembly of a computerized experimental control and recording system. Mr. Larry Gordon and Mr. Charles Lambright, who were assisting Dr. Amon in the project, and Mr. Larry Baucom, are making progress toward completion of the system. When operational, the system will make possible single or multiple subject experimentation, under precisely controlled conditions, with data acquisition at high rates recorded on magnetic tape and interpretable by the UNIVAC 1105 of the Computation Center of the University of North Carolina.

REFERENCES

- Amon, A. H. Decision structures for recognition, Report No. 126, Digital Computer Laboratory, University of Illinois, Urbana, Illinois, 1962.
- Luce, R. D. and Raiffa, H. Games and decisions. New York: Wiley & Sons, 1957.
- Messick, D. M. Bayesian decision theory, game theory, and group problem solving. Technical Documentary Report No. ESD-TDR-64-603, Decision Sciences Laboratory, Electronic Systems Division, Air Force Systems Command, United States Air Force, L. G. Hanscom Field, Bedford, Massachusetts, October 1964.
- Messick, D. M. Sequential information seeking: effects of the number of terminal acts and prior information. Technical Documentary Report No. ESD-TDR-64-606, Decision Sciences Laboratory, Electronic Systems Division, Air Force Systems Command, United States Air Force, L. G. Hanscom Field, Bedford, Massachusetts, October 1964.
- Messick, D. M. Sequential information seeking: an optimal strategy and other results. Technical Documentary Report No. ESD-TDR-64-605, Decision Sciences Laboratory, Electronic Systems Division, Air Force Systems Command, United States Air Force, L. G. Hanscom Field, Bedford, Massachusetts, October 1964.
- Raiffa, H. and Schlaifer, R. Applied statistical decision theory. Boston: Harvard Business School, 1961.
- Rapoport, Amnon. Estimation of continuous subjective probability distributions in a sequential decision task. Technical Documentary Report No. ESD-TDR-64-607, Decision Sciences Laboratory, Electronic Systems Division, Air Force Systems Command, United States Air Force, L. G. Hanscom Field, Bedford, Massachusetts, October 1964.
- Rapoport, Amnon. A study of human decisions in a computer-controlled task. Technical Documentary Report No. ESD-TDR-64-604, Decision Sciences Laboratory, Electronic Systems Division, Air Force Systems Command, United States Air Force, L. G. Hanscom Field, Bedford, Massachusetts, October 1964. (Published as "Sequential decision making in a computer-controlled task" in J. math. Psychol., Vol. 1, No. 2, July, 1964, 351-374).

Schlaifer, R. Probability and statistics in business decisions.
New York: McGraw-Hill, 1959.

Schlaifer, R. Introduction to probability and statistics. New
York: McGraw-Hill, 1961.

Shuford, E. H. A comparison of subjective probabilities for ele-
mentary and compound events. UNC Psychometric Laboratory
Report No. 20, Chapel Hill, North Carolina, January, 1959.

Shuford, E. H. and Wiesen, R. A. Bayes estimation of proportion:
the effect of stimulus distribution and exposure time. UNC
Psychometric Laboratory Report No. 23, Chapel Hill, North
Carolina, December 1959.

Shuford, E. H., Jones, L. V., and Bock, R. D. A rational origin
obtained by the method of contingent paired comparisons,
Psychometrika, 1960, 25, 343-356.

Toda, M. Measurement of subjective probability distribution.
Report No. 3, Division of Mathematical Psychology, Institute
for Research, State College, Pennsylvania.

von Neumann, J. and Morgenstern, O. Theory of games and economic
behavior. Princeton: Princeton University, 1947.

Wiesen, R. A. Decision-theoretic and empirical investigation of
some probabilistic discrimination learning situations. Technical
Documentary Report No. ESD-TDR-64-192, Decision Sciences
Laboratory, Electronic Systems Division, Air Force Systems
Command, United States Air Force, L. G. Hanscom Field, Bed-
ford, Massachusetts, January 1964.

Wiesen, R. A. and Shuford, E. H. Bayes strategies as adaptive
behavior. In E. E. Bernard and M. R. Kare (Eds.) Biological
prototypes and synthetic systems. Vol. 1, New York: Plenum
Press, 1962.

WORKING PAPERS ON DECISION THEORY

Chapel Hill Series

Volume I

No.	Date	Author	Title
1	9/27/62	Toda	RANGE ESTIMATION AS A METHOD OF MEASURING PRIOR PROBABILITY DISTRIBUTION
2	9/29/62	Toda	RANGE ESTIMATION METHOD--II
3	10/1/62	Toda	ON RANGE BETTING AND PARTITION BETTING METHODS--REPORT III
4	10/1/62	Shuford	PROPOSED EXPERIMENT TESTING BAYESIAN MODEL AND RESPONSE DISTRIBUTIONS
5	10/2/62	Shuford	LEARNING PRIOR PROBABILITIES
6	10/2/62	Toda	RANGE BETTING METHOD--REPORT NO. 4: SOME ADDITIONAL REMARKS ON THE FORMULATION OF PAYOFF FUNCTION AND ON DISCRETE CASES
7	10/7/62	Messick	THREE METHODS OF ESTIMATING CONDITIONAL PROBABILITIES
8	10/9/62	Wiesen	LEARNING PRIOR PROBABILITIES
9	10/4/62	Toda	A PROBLEM: BAYESIAN VS. MINIMAX STRATEGIES IN ZERO-SUM TWO-PERSON GAMES
10	10/19/62	Wiesen	LEARNING PRIOR PROBABILITIES

WORKING PAPERS ON DECISION THEORY

Chapel Hill Series

Volume II

No.	Date	Author	Title
11	11/19/62	Rapoport	COMPUTER-CONTROLLED DECISION TASK I
12	11/ 5/62	Massengill	COMPUTER-CONTROLLED DECISION TASK II: INSTRUCTIONS
13	11/17/62	Nakahara	A STEP-FUNCTION THEOREM CON- CERNING EXPECTED GAIN IN PROBA- BILITY LEARNING
14	11/19/62	Shuford	A DECISION-MAKING AID
15	11/19/62	Shuford	AN EXERCISE IN DEGRADATION: INTRODUCTION AND DESCRIPTION OF CASE 0
16	11/20/62	Massengill	TEACHING OPTIMAL BEHAVIOR
17	11/25/62	Shuford	AN EXERCISE IN DEGRADATION: DESCRIPTION OF CASE 1
18	11/28/62	Wiesen	LEARNING PRIOR VS. LEARNING POSTERIOR PROBABILITIES
19	11/28/62	Shuford	AN EXERCISE IN DEGRADATION: DESCRIPTION OF CASE 2

WORKING PAPERS ON DECISION THEORY

Chapel Hill Series

Volume III

No.	Date	Author	Title
20	11/30/62	Rapoport	COMPUTER-CONTROLLED DECISION TASK I: INFERRING THE PARAMETERS OF A SUBJECTIVE DISTRIBUTION
21	12/ 2/62	Shuford	AN EXERCISE IN DEGRADATION: DESCRIPTION OF CASE 3
22	12/ 6/62	Wiesen	BAYES ESTIMATION OF PROPORTIONS: THE EFFECTS OF COMPLETE AND PARTIAL FEEDBACK
23	12/ 6/62	Wiesen	A NEW STATISTICAL PROCEDURE
24	11/30/62	Messick	THE WORTH OF INFORMATION IN SOME TWO-PERSON NON-ZERO SUM GAMES
25	12/14/62	Toda and Nakahara	PILOT EXPERIMENTS WITHIN SOME SIMPLE FUNGUS-EATER CONTEXTS
26	12/17/62	Rapoport	COMPUTER-CONTROLLED DECISION TASK I: PROGRAM DESCRIPTION

WORKING PAPERS ON DECISION THEORY

Chapel Hill Series

Volume IV

No.	Date	Author	Title
27	1/ 4/63	Messick	EXPERIMENTAL DESIGN TO INVESTIGATE EFFECTS OF DECISION CATEGORIES ON PREDECISION INFORMATION SEEKING
28	1/ 4/63	Nishisato	TIME-ORDER-ERROR EXPERIMENT: METHOD AND RESULTS
29	1/ 6/63	Wiesen	REPLICATION REPORT: REMEMBERING THE PRESENT STATES OF A NUMBER OF VARIABLES
30	1/ 6/63	Wiesen	A RESPONSE-BIAS ANALYSIS OF TIME-ORDER ERRORS
31	1/ 8/63	Shuford	AN EXERCISE IN DEGRADATION: DESCRIPTION OF CASE 4
32	1/10/63	Toda	BOOK REVIEW: <u>UNCERTAINTY AND STRUCTURE AS PSYCHOLOGICAL CONCEPTS.</u> WENDELL R. GARNER. JOHN WILEY AND SONS, INC.: NEW YORK, 1962, 369 PP.

III. ABSTRACTS OF TECHNICAL DOCUMENTARY REPORTS

ESD-TDR-64-603

David M. Messick

BAYESIAN DECISION THEORY, GAME THEORY AND GROUP PROBLEM-SOLVING

This paper applies the concepts of the "ideal" information processor and rational decision maker to a typical problem in social psychology--that of group problem solving. The structure of the selected task is seen to be that of a nonzero sum game. A strategy is derived and is shown to be the equivalent of the Nash solution to the game. The notion of level of aspiration is discussed and defined within the analysis and two theorems are proved relating level of aspiration to type of group decision strategy employed.

Project & Task: 4690-469003

Contract No. AF 19(628)-1610

ESD-TDR-64-605

David M. Messick

SEQUENTIAL INFORMATION SEEKING: AN OPTIMAL STRATEGY AND OTHER RESULTS

This paper presents an optimal strategy for sequential sampling from binomial distributions. The strategy presented is general in that it is a "multi-action" rather than a two-action procedure. While the major task is to estimate the proportion, p , of "successes" in a hypothetical, infinite population of binary observations, it is assumed that the decision maker is only concerned with which of a set of mutually exclusive and exhaustive subsets of the unit interval contains p . The derived strategy maximizes the decision-maker's gain without regard to error probabilities.

The important variable in determining a rule for ceasing to look at new data and making a decision is found to be the expected probability of being correct. The criterion involves only the economic aspects of the situation. A "no information" theorer is presented which shows that under some circumstances when a "success" or a "failure" on a given trial are equally probable, the probability of being correct after making observation is identical to the probability of being correct before the observation was taken. Finally, an appealing derivation of the Beta-binomial probability function is given which suggests a more tractable computational procedure for the distribution and which illuminates its limiting distribution.

Project & Task: 4690-469003

Contract No. AF 19(628)-1610

David M. Messick

SEQUENTIAL INFORMATION SEEKING · EFFECTS OF THE NUMBER OF TERMINAL ACTS AND PRIOR INFORMATION

An experiment was conducted by means of a digital computer in which 54 human Ss were faced with the task of sampling from a hypothetical binomial universe in which a proportion, p , of all observations were "top quality." Ss sampled the universe sequentially, stopping after some number of observations had been made to make a terminal decision by selecting the one of the S's mutually exclusive and exhaustive subsets of the unit interval which S believed contained p . The four experimental treatments were defined by the four combinations of the two decision partitions of the unit interval, one involving 3 possible terminal acts, the other having 5 alternatives, and the two prior frequency distributions, one a rectangular distribution over [0 - 1], the other being negatively skewed. Analysis of variance of the number of predecision observations taken indicated a) significant individual differences; b) significant S by treatment interactions; c) differences attributable to the decision partitions with more observations being taken in the 5-act case than in the 3-act case; and d) no effect of prior frequency distributions, but a tendency to take more observations in the second 16 trials than in the first 16.

Project & Task: 4690-469003

Contract No. AF 19(528)-1610

Amnon Rapoport

A STUDY OF HUMAN DECISIONS IN A COMPUTER-CONTROLLED TASK (Published as SEQUENTIAL DECISION MAKING IN A COMPUTER-CONTROLLED TASK in J. Math. Psychol., Vol. 1, No. 2, 351-374, July, 1964).

Twenty-four undergraduate students participated in a computer-controlled experimental task, intended as an investigation of decision-making processes in a nonstationary environment. The subjects, who were tested individually, were instructed to take a sequence of predictions and estimates conditional upon typed information provided by the LGP-30 computer. Specifically, each subject was required to predict the value of an unknown p , $0 \leq p \leq 1.00$, and then to estimate its values after obtaining partial information about it from the computer. The procedure was repeated for 120 values of p , generated randomly by three different beta distributions. Subjects were paid immediately according to their success. The parameters of the subjective probability distributions were inferred from the subject's predictions and estimates in a block of ten trials. In addition, direct measures of the subjective probability distributions were obtained for every block from the subject's verbal responses given to three hypothetical bets concerning the value of the next p .

An optimal model, derived from Bayesian decision theory, was tested and partially confirmed by the subjects' responses. Alternative models were derived and tested, and some theoretical implications concerning learning processes and the use of computers in psychological experiments were discussed briefly.

Project & Task: 4690-469003

Contract No. AF 19(628)-1610

Amnon Rapoport

ESTIMATION OF CONTINUOUS SUBJECTIVE PROBABILITY DISTRIBUTION IN A
SEQUENTIAL DECISION TASK

A range betting method (RBM) for measuring continuous subjective probability distributions is presented. Predictions derived from the method are tested in a computer-controlled sequential decision making task. In particular, twenty-two undergraduate students participated individually in the task. The subjects were asked occasionally to place a wager upon the range of an unknown value of P , $0 \leq P \leq 1$, generated by an LGP-30 computer. In addition, the subjects predicted the exact value of P by typing a number on a Flexowriter, and then estimated the value of P after obtaining information about its value. The information was given in the form of n values ($1 \leq n \leq 20$) of 0's or 1's generated by a binomial distribution with mean P and printed on the Flexowriter in front of the subject.

Predicted decisions were derived both from the RBM method and from a relatively simple confidence limits' model. The subjects' estimates of the range boundaries were compared to the predicted values from both models. In both comparisons the results indicated a regression effect and a block-to-block tracking behavior. Attempts to compare two independent measures of the same subjective probability distribution were unsuccessful.

Project & Task: 4690-469003

Contract No. AF 19(628)-1610

Raymond A. Wiesen

DECISION-THEORETIC AND EMPIRICAL INVESTIGATION OF SOME PROBABILISTIC DISCRIMINATION LEARNING SITUATIONS

A decision-theoretic analysis and experiment of three related choice situations is presented. The first situation is a standard probabilistic discrimination learning task. Each task begins with the presentation of one of a set of stimuli. The subject must choose between two response alternatives to predict which of the two events will occur on the trial, the probability of each event being a function of the stimulus presented. The second situation arises when the conditional probabilities, i.e., the probabilities of the stimuli given the events, are introduced to the subject at the beginning of the experiment. The third situation is like the second except for the fact that the subject is not told which event occurs on each trial.

The decision-theoretic analysis shows what differences in performance would be expected among the three conditions when a strategy which maximizes average expected payoff is employed.

One group of subjects was run in each situation with the overall relative frequency of one event equal to .80. The performance of the subjects in the first and second situations was virtually identical, while the performance of the subjects in the third (non-feedback) was somewhat worse. The performance measure was the sum of the differences between the objective expected payoff of the optimal choices and the choices made by the subject. Comparisons of the choice proportions for the first and second groups indicated that subjects in the second group did not integrate information concerning the overall relative frequencies of events and conditional probabilities. A large proportion of subjects in the third (non-feedback) group made every choice in agreement with the assumption that the overall relative frequency of one event was one-half.

Project & Task: 4690-469003

Contract No. AF 19(628)-1610